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Growth of a Dissipative Universe and the Limits of Newtonian Cosmology

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In closed model universes assumed to bounce at some minimum radius, dissipation causes the amplitude of the oscillations to grow. This well-known fact of relativistic cosmology is counterintuitive. Since Newtonian models correspond closely to relativistic ones when there is no dissipation, we examine dissipative Newtonian models, and note the peculiarly relativistic ideas which must be introduced to obtain correct results.

1. INTRODUCTION

It has been known since Tolman's (1934) work that dissipative processes cause the oscillations of a closed model universe which bounces at a minimum radius to grow in amplitude from one cycle to the next. This is contrary to our experience that dissipative processes, such as those involved in the damping of a harmonic oscillator, lead to a *decrease* in amplitude.

Homogeneous Newtonian models *without* dissipation correspond closely to those of general relativity. This is of pedagogic value, for one can discuss basic cosmological models rigorously without the mechanism of Einstein's theory. Here we consider dissipative Newtonian models, discern their limits, and point out the peculiarly relativistic concepts which lead to Tolman's result. This shows where classical intuition goes astray, and makes it possible to present Tolman's result to students familiar only with Newtonian mechanics and special relativity.

2. NEWTONIAN AND RELATIVISTIC COSMOLOGY

Einstein's theory allows three types of homogeneous and isotropic model universes, flat and positively or negatively curved, and relates the

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time dependence of the cosmic scale factor to the matter content. It has been known for a long time that cosmologies based on Newtonian theory give results very similar to those of general relativity. This is no accident, but a rigorous consequence of Einstein's theory itself (Callan *et al.*, 1965). For a homogeneous universe containing only pressure-free dust, we may focus on a spherical portion so small and containing so little mass that it can be described by Euclidean geometry and Newtonian physics. We obtain an equation which governs the change of the sphere's radius with time. This yields a scale factor which will, at any time, give the distance between two arbitrary particles in the sphere. But since the model is homogeneous, the same scale factor gives the distance between *any* two particles in the entire universe as a function of time. The equation governing the dynamics of the small sphere determines the dynamics of the cosmos as a whole.

If the radius of the sphere is $a(t)$ and the constant mass within it M, a test particle on its surface will move in accord with the Newtonian energy equation

$$(1/2)(da/dt)^2 - GM/a = W = \text{const} \tag{1}$$

The integration constant W, the particle's energy divided by its mass, will be critical in our discussion. It would be arbitrary for a finite gas sphere, but not for an entire universe. If the displacement between any two particles at the instant when $a = 1$ is $\mathbf{r}$, that at time t will be $a(t)\mathbf{r}$. We will consider the vector $\mathbf{r}$ to have dimensionless components, so that a will continue to have units of length (McVittie, 1965). $a(t)$ specifies the state of the universe, and cannot depend on a property of only *part* of it such as M, but solely on the cosmic time t, the uniform density, and universal constants. W, which enters into the solution for $a(t)$, must be expressible in terms of a fixed value of the cosmic time t_0, the density ρ_0 at that time, and the gravitational constant G, since those are the only constants in the Newtonian formulation of the problem. But an equation of the form $W = t_0^\alpha \rho_0^\beta G^\gamma$ cannot be dimensionally correct. W is therefore an *independent* universal constant with the dimensions of a squared speed, but we cannot say more than this within the confines of Newtonian theory. [This argument may be compared with one in Milne's (1948) theory.]

Thus Newtonian theory is incomplete. It does not require much familiarity with relativity to guess that W must be proportional to c^2 to give the correct correspondence. In fact, Einstein's theory yields a result very similar to (1), the Friedman equation:

$$(1/2)(da/dt)^2 - GM/a = -kc^2/2 \tag{2}$$

By an appropriate choice of units, the value of the constant k can be made to be +1, −1, or 0. The corresponding solutions represent, respectively, spaces of constant positive or negative curvature or flat space. In the first case, the universe expands to a maximum size and then contracts, while $a(t)$ always increases for $k = -1$ or 0. In Newtonian theory, these are the same types of behavior one could have in shooting a rocket away from a planet with an initial speed less than, greater than, or equal to escape speed.

We are interested in spherical spaces for which the solution of (2) is

$$\begin{aligned} a &= (GM/c^2)(1 - \cos\psi) \\ t &= (GM/c^3)(\psi - \sin\psi) \end{aligned} \tag{3}$$

which are the parametric equations of a cycloid. Here M is an abbreviation for $4\pi\rho_0 a_0{}^3/3$, quantities being evaluated at some arbitrary time t_0. As we have seen, the solution cannot involve properties peculiar to a limited portion of the universe.

The periodic character of (3) has led to speculation about eternally oscillating universes. One difficulty with such ideas is the existence of space-time singularities at points given by the cusps of the cycloid. More precisely, the singular "points" with $t = 2n\pi GM/c^3$ and $a = 0$ do not *exist*: The space-time manifold is incomplete (Hawking and Ellis, 1973).

Modifications of Einstein's classical theory of gravitation, such as quantization, may change the solution near the cusps, so that the universe bounces from a contracting to an expanding phase with no singularity. The nature of such a bounce is not our concern here, and we simply assume that it can take place and that the basic parameters of physics (e.g., G) are not changed in the bounce, so that we can talk meaningfully about an oscillating universe.

There is a Newtonian analogy for such a model. We noted that the dynamics of a universe with $W < 0$ corresponds to the motion of an object projected at less than escape speed from the surface of a planet. If, on return, it collides elastically with the planet's surface, the resulting motion would be like that of a model universe which bounces at some minimum radius.

3. CLOSED UNIVERSES WITH DISSIPATION

Matters are not so simple when dissipation is present. Tolman showed that the behavior of a closed universe is changed significantly, and in a way contrary to classical intuition, when processes which are not isentropic take place within it. Such processes must be taken into account if our models are to be at all realistic, and especially if we want to discuss the possibility of eternally oscillating universes.

The effect of irreversible thermodynamic processes on cosmological models has been discussed by a number of authors (e.g., Tolman, 1934; Treciokas and Ellis, 1971; Nightingale, 1973; Landsberg and Park, 1975; Neugebauer and Meier, 1976). A major result is that, in closed universes of the type we have been considering, there is a secular change in the amplitude of the a versus t curve from one cycle to the next. But this does not happen in the way one would first expect. A bouncing ball will rise to the same height on each bounce if its collisions with the ground are elastic, but the height will decrease if energy is dissipated in the collisions. Similarly, one would expect that the amplitude of cosmic oscillations would be damped out. But this is not the case. The amplitude actually increases. A closed universe in which dissipative processes take place grows with time.

Why does this counterintuitive behavior occur? Since it is possible to derive the equations describing nondissipative models in a rigorous way from Newtonian theory, we might hope that this could also be the case with dissipation. This hope cannot be fulfilled completely because relativistic considerations are essential at a couple of points. We have already noted that relativity is needed to give the value of W in (1). We must also go beyond Newtonian concepts to recognize that *all* energy has mass, and acts as a source of gravitational fields. Thus (2) can be written

$$(da/dt)^2 - 2GE/c^2a + kc^2 = 0 \tag{4}$$

with E the total energy within a sphere of radius a.

The first and second laws of thermodynamics for pressure-free dust with no chemical reactions give $dE = T\,dS$. If $dS > 0$, then $dE > 0$, so a monotonic increase in entropy means that E, and thus the active gravitational mass, will increase.

The simplest example of the processes we are considering is bulk viscosity from transfer of energy between different microscopic degrees of freedom (Landau and Lifshitz, 1959). It will be effective in pure expansion or contraction of a fluid, even with no shear, and has the effect of a *negative* pressure in the Einstein equations. In expansion, this negative pressure does negative work, and thus creates energy. A model in which this effect is dominant exhibits the de Sitter expansion of the steady-state or inflationary cosmologies (McCrea, 1951; Murphy, 1973; Guth, 1980).

In our universe, heat conduction and shear viscosity are more important than bulk viscosity. But the latter provides a simple way of averaging the net thermodynamic and gravitational effects of heat conduction or shear viscosity without having to take into account explicitly the inhomogeneities or anisotropies which give rise to them.

All such processes affect the expansion of the universe. In (4), E is now a monotonically increasing function of time, and we take $k = +1$. If t_1 and

t_2 are successive times at which $da/dt = 0$, so that the expansion has reached a local maximum, then, since $E_2 > E_1$, $a_2 = 2GE_2/c^4 > 2GE_1/c^4 = a_1$. Thus the maximum scale factor will reach successively larger and larger values. This is Tolman's result. Note also that, in each cycle, the curve of a versus t is steeper during contraction than in the preceding expansion: For a given value of a, $(da/dt)^2$ is greater after the maximum is reached than before.

It is important to note the role of k. It corresponds to the Newtonian energy per unit mass of a particle on the edge of our small sphere, but is not directly related to the total energy E inside the sphere. Pure Newtonian theory is misleading when we consider dissipative universes. k, and hence W, do not change, as would the Newtonian energy of a planet orbiting a star whose mass grows by accretion.

There cannot, then, be a strict recurrence of cosmic states when in the presence of dissipation. It is even possible for a model universe after a certain number of oscillations to "take off" on a final expansion and never recontract. This can be shown for the case of bulk viscosity, which produces an equivalent pressure $p' = -\zeta(1/V)\, dV/dt$, ζ being the bulk viscosity coefficient and V the fluid volume. Then

$$dE/dt = 12\pi\zeta a(da/dt)^2 \tag{5}$$

is the rate of change of internal energy. Solution of (4) and (5) together will give E and a as functions of time. Neugebauer and Meier (1976) transform these equations into a single nonlinear second-order equation and show that, if $\zeta > c^4/24\pi Ga(da/dt)$ at some time in an expanding phase, then a will have no finite maximum. Here we simply note that we can find an unbounded asymptotic solution of (4) and (5), valid for large values of t. Specifically, we try

$$a = A \exp(\alpha t), \qquad E = B \exp(\beta t) \tag{6}$$

with A, B, α, and β constants. For sufficiently large times, the curvature term in (4) can be neglected and we find that (4) and (5) are satisfied if $\beta = 3\alpha = 24\pi G\zeta/c^2$. The internal energy is proportional to the volume. The exponential relation between a and t is, of course, that of the de Sitter solution which obtains in the steady-state or inflationary cosmologies.

4. CONCLUSIONS

Why do dissipative universes grow, when our experience of such processes is that they damp out oscillatory motions? In idealized homogeneous universes, such processes do not act directly as friction to remove energy from large-scale motion. There is no "outside" to which energy can be removed. The increase in internal energy means an increase in active gravitational mass. This, combined with the constancy of k required by the homogeneity of the universe, leads to an increase in amplitude: E/a must be the same at each maximum, and a_{max} must increase because E does.

Philosophical and religious traditions take different positions on a continual recurrence of the world (Eliade, 1954). One may or may not feel that the results of scientific cosmology are germane to such considerations, but it seems well for those concerned about such matters to know what those results are. The approach of this paper, which remains as close as possible to that of Newtonian mechanics, should help to make these results accessible to a wider audience.

REFERENCES

Callan, C., Dicke, R. H., and Peebles, P. J. E. (1965). *American Journal of Physics*, **33**, 105.

Eliade, M. (1954). *The Myth of the Eternal Return or, Cosmos and History*, Princeton University Press, Princeton, New Jersey.

Guth, A. (1980). *Physical Review D*, **23**, 347.

Hawking, S. W., and Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time*, Cambridge University Press, Cambridge, Chapter 8.

Landau, L. D., and Lifshitz, E. M. (1959). *Fluid Mechanics*, Addison-Wesley, Reading, Massachusetts, pp. 304–309.

Landsberg, P. T., and Park, D. (1975). *Proceedings of the Royal Society A*, **346**, 485.

McCrea, W. H. (1951). *Proceedings of the Royal Society*, **206**, 562.

McVittie, G. C. (1965). *General Relativity and Cosmology*, 2nd ed., University of Illinois Press, Urbana, Illinois, p. 143.

Milne, E. A. (1948). *Kinematic Relativity*, Clarendon Press, Oxford, pp. 68–69.

Murphy, G. L. (1973). *Physical Review D*, **8**, 4231.

Neugebauer, G., and Meier, W. (1976). *Annalen der Physik*, *7*, **33**, 161.

Nightingale, J. D. (1973). *Astrophysical Journal*, **185**, 105.

Tolman, R. C. (1934). *Relativity, Thermodynamics and Cosmology*, Clarendon Press, Oxford, Section 175.

Treciokas, R., and Ellis, G. F. R. (1971). *Communications in Mathematical Physics*, **23**, 1.